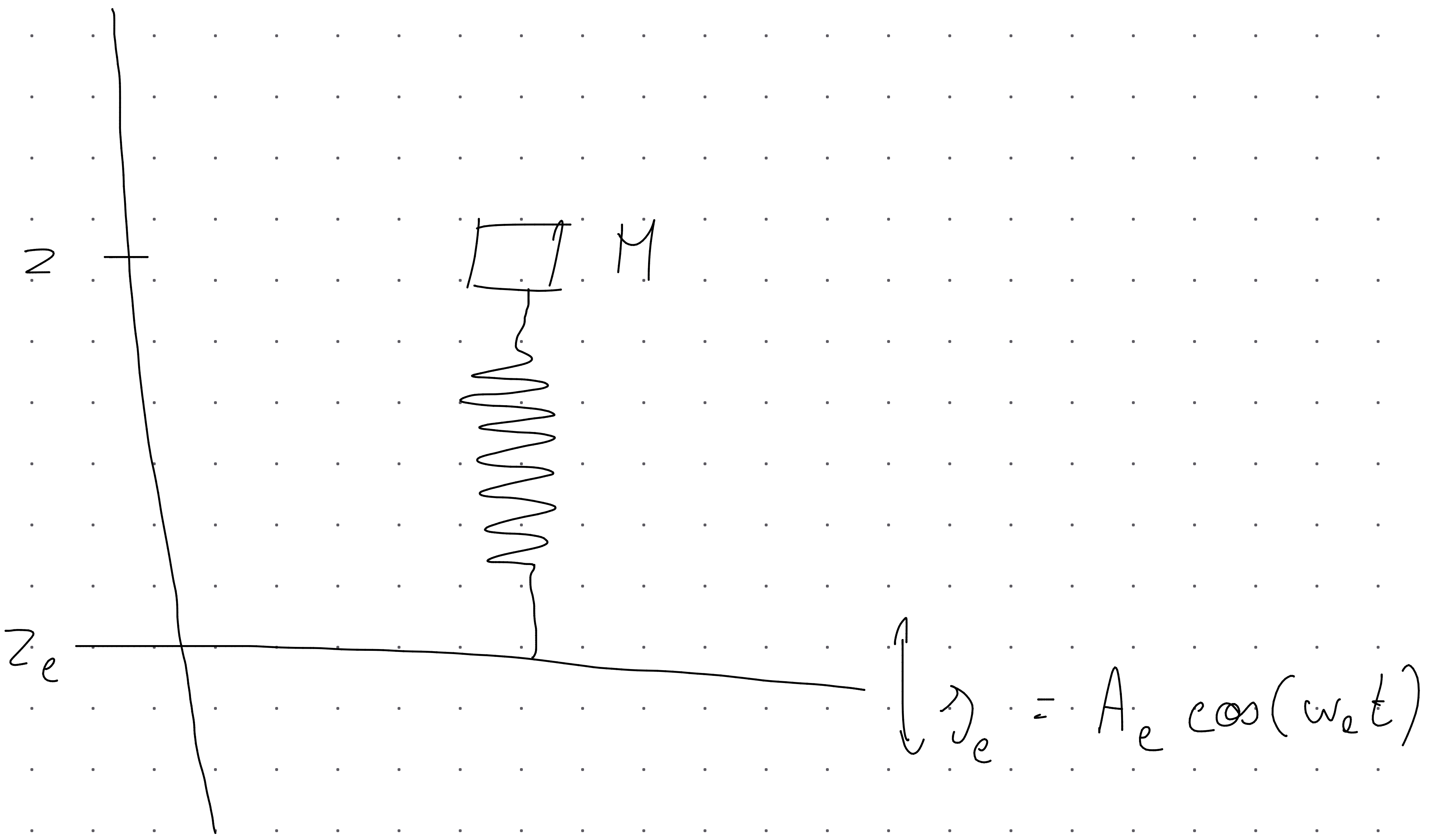
Exc 8 :



$\sum F_{ext} = k \Delta l \vec{y}$

$m\vec{a} = k(z_e - z)\vec{y}$

$m \dfrac{d^2 y}{dt^2} = k(z_e - z)$

$\Rightarrow \dfrac{d^2 y}{dt^2} = \dfrac{k z_e}{m} - \dfrac{k z}{m} = A_e \cos(\omega_e t)$

Solution homogène : (Sans second membre)

$\dfrac{dy^2}{dt^2} + \dfrac{k z}{m} = 0$ on pose $\omega_0 = \sqrt{\dfrac{k}{m}}$

$y_h = A\cos(\omega_0 t + \varphi)$

Solution particulière

$$y_p = A_p \cos(\omega_e t + \phi)$$
$$= B_p \sin(\omega_e t + \beta)$$
$$= C_p \cos(\omega_e t) + D_p \sin(\omega_e t)$$

$$\frac{d^2 y_p}{dt^2} = -\omega_e^2 y_p$$

$$-\omega_e^2 y_p + \omega^2 y_p = \frac{K A_e}{m} \mathrm{Re}\left[e^{j\omega_e t}\right]$$

$$-\omega_e^2 A_p e^{j(\omega_e t + \phi)} + \omega_0^2 A_p e^{j(\omega_e t + \phi)} = \frac{K A_e}{m} e^{j\omega_e t}$$

$$A_p(\omega_0^2 - \omega_e^2) e^{j\phi} = \frac{A_e}{m} K$$

$$A_p e^{i\phi} = \frac{A_e K}{m(\omega_0^2 - \omega_e^2)} = \frac{\omega_0^2}{\omega_0^2 - \omega_e^2} A_e$$

$$\Rightarrow \phi = 0 \text{ et } A_p = \frac{A_e}{1 - \left(\frac{\omega_e}{\omega_0}\right)^2}$$

$$\boxed{y_p = \frac{A_e}{1 - \left(\frac{\omega_e}{\omega_0}\right)^2} \cos(\omega_e t)}$$

$$y(t) = A\cos(\omega_0 t + \varphi) + \frac{A_e}{1 + \left(\frac{\omega_e}{\omega_0}\right)^2} \cos(\omega_e t)$$

$$\lim_{\omega_e \to \omega_0} \frac{A_e}{1 - \left(\frac{\omega_e}{\omega_0}\right)^2} = +\infty$$ donc $y(t)$ admet un maximum.

La fréquence de résonance est ω_0

4) $A_p = \left| \dfrac{A_e}{1-\left(\dfrac{\omega_e}{\omega_0}\right)^2} \right| = \left| \dfrac{1}{100} A_e \right|$

si $\omega_e < \omega_0 : 1-\left(\dfrac{\omega_e}{\omega_0}\right)^2 = 100$

$\omega_e > \omega_0 : \left(\dfrac{\omega_e}{\omega_0}\right)^2 - 1 = 100$

$\omega_0 = \dfrac{\omega_e}{\sqrt{101}}$

$\omega_0 = 12{,}5 \text{ rd/s}$

5) $K\Delta l = Mg$

$\Delta l = \dfrac{Mg}{K} \Leftrightarrow \Delta z = \dfrac{Mg}{K} \Leftrightarrow \Delta z = \dfrac{g}{\omega_0^2} = \dfrac{9{,}81}{(12{,}5)^2} = 6{,}3 \text{ cm}$